FREQUENCY CHARACTERISTICS OF A LAMINAR FLAME

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A theoretical study is made of the effect of harmonic pressure oscillations on a flat laminar flame. Frequency characteristics of the flames are obtained at amplitudes of the pressure oscillations which are small compared with the average pressure. It is established that in a broad range of frequencies the disturbances in the integral rate of heat release occur in phase with the pressure oscillations and depend weakly on the frequency.

1. The stability relative to acoustical vibrations of the gas of a system in which combustion occurs depends in many cases on the reaction of the flame to the pressure disturbances.

Using the apparatus of the theory of automatic regulation, such a response (reaction) can be described by the frequency characteristics of the flame. When the size of the combustion zone is much less than the length of the sonic wave, an important frequency characteristic of the flame is the value $G(\omega) = (S'/S_0)/(p'/p_0)$, where $G(\omega)$ represents a value analogous to the "interaction index" of Crocco [1]. It determines the reaction of the integral rate of heat release S in the combustion zone to the harmonic pressure disturbances p' of different frequency ω

$$S = Q \int_{-\infty}^{+\infty} W(x)\, dx$$

where W(x) and Q are the volumetric rate and calorific effect of the chemical reaction, S_0 and p_0 are average values, and S' is the disturbance.

The frequency characteristics of flames are poorly studied. They are rarely introduced into theoretical constructions based on intuitive concepts [2, 3]. The purpose of the present work is to determine the frequency characteristic $G(\omega)$ for a flat model laminar flame front, whose behavior is described by the thermal theory of Zel'dovich and Frank-Kamenetskii [4]. The problem is solved numerically using an electronic computer. The characteristic $G(\omega)$ can be found by two methods: by direct integration of the original equations with harmonic pressure disturbances and using transition functions. The statement of the problem for each of these methods is given below.

2. The propagation of a homogeneous laminar flame front is described by the system of equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0, \quad \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x}$$

$$c_p \rho \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x}\right) = \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\chi \frac{\partial T}{\partial x}\right) + QW \qquad (2.1)$$

$$\rho \left(\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x}\right) = \frac{\partial}{\partial x}\left(\rho D \frac{\partial h}{\partial x}\right) + W, \quad p = \frac{\rho}{\mu} RT$$

$$W = k_i \rho^{\nu} (1 - h)^{\nu} \exp(-E/RT) \qquad (2.2)$$

Here ρ is the density, u is the velocity, p is the pressure, T is the gas temperature, h is the relative weight concentration of the combustion products or the completeness of conversion, χ is the thermal conductivity coefficient, D is the diffusion coefficient, μ is the molecular weight of the components, R is the

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universal gas constant, c_p is the specific heat capacity at constant pressure, W is the reaction rate, E is the activation energy, ν is the order of the chemical reaction (in the present work $\nu = 1$ or 2), and k_i is a preexponential factor corresponding to the order of the reaction, $i = \nu$.

The Eqs. (2.1) are valid on the assumptions that the specific heat capacities of the components of the mixture are constant and equal, the transport coefficients are independent of the composition, the molecular weights of the initial mixture and the combustion products are equal, and the effect of viscosity, thermo- and barodiffusion, and radiant heat transport is neglected.

To simplify system (2.1) let us convert to the mass Lagrangian coordinates

$$q = \int_0^x \rho(x, t)\, dx \quad (x \geqslant 0), \quad t_q = t$$

and the dimensionless variables

$$\tau = \frac{t}{t_0}, \quad \xi = \frac{q}{q_0}, \quad \vartheta = \frac{(T - T_{20})E}{RT_{20}^2}, \quad \sigma = \frac{h - h_{10}}{1 - h_{10}}$$
$$\bar{u} = \frac{u}{u_0}, \quad \bar{\rho} = \frac{\rho}{\rho_{10}}, \quad \bar{p} = \frac{p}{p_{10}}, \quad \bar{S} = \frac{S}{S_0}$$

Here the subscript 0 corresponds to the stationary mode of propagation of the flame in the absence of pressure disturbances, 1 corresponds to the initial mixture, and 2 to the combustion products. A dash on top indicates that the value under consideration is dimensionless. The relationship between the parameters t_0, q_0, and u_0 is given by the Eqs. (2.3)

$$q_0 = \chi_{10} / c_p u_0, \quad t_0 = \chi_{10} / \rho_{10} c_p u_0^2 \tag{2.3}$$

On the basis of the theory of Zel'dovich and Frank-Kamenetskii it is convenient to take u_0 in the form

$$u_0^2 = \frac{\chi_{10} k_i \rho_{10}^{\nu-2} (1 - h_{10})^{\nu-1}}{c_p (T_{20} - T_{10})^{\nu-1}} \left(\frac{T_{10}}{T_{20}}\right)^{\nu} \left(\frac{RT_{20}^2}{E}\right)^{\nu+1} \exp\left(-\frac{E}{RT_{20}}\right) \tag{2.4}$$

Neglecting the weak dependence of $\rho\chi$ and $\rho^2 D$ on the temperature ($\rho\chi \sim \rho^2 D \sim T^{-0.25}$) and using the equation of state, the condition of thermal balance $Q = c_p(T_{20} - T_{10})\ (1 - h_{10})$, and the equation $D_1\rho_1 = D_{10}\rho_{10}$, we obtain

$$\partial u / \partial \xi = \partial / \partial \tau\ (1 / \rho) \tag{2.5}$$

$$P_0 \partial u / \partial \tau = -\partial p / \partial \xi \tag{2.6}$$

$$\frac{\partial \vartheta}{\partial \tau} - \Gamma(\vartheta + g_0)\frac{1}{p}\frac{\partial p}{\partial \tau} = \rho_1 \frac{\partial^2 \vartheta}{\partial \xi^2} + F \tag{2.7}$$

$$\partial \sigma / \partial \tau = \mathrm{Le}\, \rho_1 \partial^2 \sigma / \partial \xi^2 - F / \vartheta_{10} \tag{2.8}$$

$$p = \rho\ (\vartheta + g_0) / (\vartheta_{10} + g_0) \tag{2.9}$$

$$F = (-1)^{\nu} \vartheta_{10}^{\nu+2} \alpha_0^{\nu} p_1^{\nu-1} \left(\frac{\vartheta_{10} + g_0}{\vartheta + g_0}\right)^{\nu-1} (1 - \sigma)^{\nu} \exp\left(\frac{\vartheta g_0}{\vartheta + g_0}\right)$$
$$P_0 = \frac{\rho_{10} u_0^2}{p_{10}}, \quad \Gamma = \frac{R}{\mu c_p} = \frac{\gamma - 1}{\gamma}, \quad \gamma = \frac{c_p}{c_V}, \quad \vartheta_{10} = \frac{(T_{10} - T_{20})E}{RT_{20}^2} \tag{2.10}$$
$$\mathrm{Le} = \frac{\rho_{10} c_p D_{10}}{\chi_{10}}, \quad g_0 = \frac{E}{RT_{20}}, \quad \alpha_0 = \frac{T_{20}}{T_{10}}$$

Here and later we omit the dashes above the dimensionless variables; c_V is the specific heat capacity at constant volume.

In the variables selected the relative disturbance in the integral rate of heat release, which characterizes the response of the flame to the pressure disturbances, is

$$S' = \int_0^{\infty} (F(\xi, \tau) - F_0(\xi))\, d\xi \Big/ \int_0^{\infty} F_0(\xi)\, d\xi \tag{2.11}$$

The term containing $\partial p/\partial\tau$ in Eq. (2.7) allows for the pressure variation in the flame front and the periodic pressure oscillations in the gas. Since the kinetic energy of the gas motion during the propagation of the flame is much less than its thermal energy we will neglect the component which allows for the pressure variation in the flame front. We retain the component connected with the periodic pressure oscillations since it is essential for the description of the flame propagation in a variable pressure field. The length of the sound wave is assumed to be much greater than the width of the flame front, so the pressure disturbance is taken as identical over the entire space. Only the density of the initial mixture, connected with the pressure by the isentropic equation $\rho_1 = \rho_1^{1/\gamma}$, enters into Eqs. (2.7) and (2.8). Consequently they can be solved independently from Eqs. (2.5), (2.6), and (2.9).

Taking the pressure disturbance in the form $p' = \delta p \cos \Omega\tau$ (where δp and Ω are the dimensionless amplitude and angular frequency of the pressure oscillations, $\Omega = \omega t_0$), we take as the boundary conditions the adiabatically disturbed stationary temperature distribution $\vartheta_0(\xi)$ and the undistributed stationary distribution of the normalized completeness of conversion $\sigma_0(\xi)$. To determine the frequency characteristic of the flame one must solve the problem

$$\begin{aligned}
&\frac{\partial\vartheta}{\partial\tau} - \Gamma(\vartheta + g_0)\frac{1}{(1+p')}\frac{\partial p'}{\partial\tau} = (1+p')^{1/\gamma}\frac{\partial^2\vartheta}{\partial\xi^2} + F\\
&\frac{\partial\sigma}{\partial\tau} = \mathrm{Le}\,(1+p')^{1/\gamma}\frac{\partial^2\sigma}{\partial\xi^2} - \frac{F}{\vartheta_{10}}\\
&\vartheta(\xi) = \eta\vartheta_0(\xi) + (\eta - 1)\,g_0,\ \ \sigma(\xi) = \sigma_0(\xi) \qquad \text{for}\quad \tau = 0\\
&\vartheta(\tau) = (\eta - 1)\,g_0,\ \ \sigma(\tau) = 1 \qquad \text{for}\quad \xi = 0\\
&\vartheta(\tau) = \eta\vartheta_{10} + (\eta - 1)\,g_0,\ \ \sigma(\tau) = 0 \qquad \text{for}\quad \xi = +\infty
\end{aligned} \tag{2.12}$$

where $\eta = (1+p')^{(\gamma-1)/\gamma}$, while F is determined by Eq. (2.10).

The problem formulated describes both the stationary and the nonstationary modes of flame propagation. Because of the nonlinearity of the equations, the stationary oscillations in the integral rate of heat release prove to be anharmonic. These oscillations are expanded in a Fourier series and for the first harmonic of the oscillations, which can be represented in the form

$$S' = A(\Omega)\exp(i\Omega\tau)$$

the amplitude-phase frequency characteristic is calculated as

$$G(\Omega) = A(\Omega)/\delta p$$

This method requires large expenditures of machine time. Therefore it will only be used to verify the applicability of the more economical method of determining the frequency characteristics from transition functions to the problem of finding the response of a flame to small pressure disturbances. The latter method is developed for systems described by linear ordinary differential equations. According to this method the frequency characteristic of a system is determined with the help of a Laplace transformation from its response to a single jump in the input parameter (i.e., from its transition function) [5].

$$G(\Omega) = H(+0) + \int_0^\infty \frac{d\Psi(\tau)}{d\tau}\exp(-i\Omega\tau)\,d\tau \tag{2.13}$$

Here $H(\tau)$ is the transition function connected with $S(\tau)$ by the equation $H(\tau) = S'/\Delta p$, S' is determined by Eq. (2.11), Δp is the size of the pressure jump, and $\psi(\tau) = H(\tau) - H(+0)$.

Equation (2.13) assumes that the principle of superposition is valid for the disturbances. Therefore the present method is applicable only for small deviations from the stationary mode of flame propagation, i.e., for small pressure disturbances. The allowable values of the pressure disturbances are not known beforehand and are determined as a result of the numerical calculations by increasing the size of the pressure disturbance to the point where marked changes develop in the form of the amplitude-phase frequency characteristic.

To determine the transition function let us examine the behavior of the flame front when $\tau > 0$ after application of the pressure jump $p' = \Delta p\, e(\tau)$, where

$$e(\tau) = \begin{cases} 1 & \text{for } \tau \geqslant 0 \\ 0 & \text{for } \tau < 0 \end{cases}$$

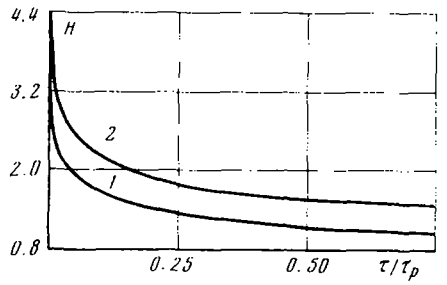

Fig. 1

The arrival at the new stationary mode of flame propagation is described by the equations

$$\frac{\partial \vartheta}{\partial \tau} = (1+\Delta p)^{1/\gamma}\frac{\partial^2 \vartheta}{\partial \xi^2} + F, \quad \frac{\partial \sigma}{\partial \tau} = \mathrm{Le}\,(1+\Delta p)^{1/\gamma}\frac{\partial^2 \sigma}{\partial \xi^2} - \frac{F}{\vartheta_{10}} \quad (2.14)$$

The boundary conditions are taken analogously with the first method [see Eq. (2.12)].

According to Eq. (2.13), to determine $G(\Omega)$ it is sufficient to know the behavior of $H(\tau)$ up to the moment that the stationary mode of flame propagation is established. The transitional process was considered as ended when $d \ln H(\tau)/d\tau \leq 0.01$.

The stationary distributions of temperature $\vartheta_0(\xi)$ and completeness of conversion $\sigma_0(\xi)$, which are used to set the boundary conditions (2.12), are found analogously to [6] by numerical integration of Eqs. (2.14) with $\Delta p = 0$. The initial conditions for the temperature and completeness of conversion are taken in step form for $\tau = 0$

$$\vartheta(\xi) = \begin{cases} \vartheta_{11} & \text{for } \xi \geqslant \xi_1 \\ 0 & \text{for } \xi < \xi_1 \end{cases}, \quad \sigma(\xi) = \begin{cases} 0 & \text{for } \xi \geqslant \xi_1 \\ 1 & \text{for } \xi < \xi_1 \end{cases}$$

where ξ_1 is the coordinate of an arbitrary point sufficiently far from the origin of coordinates, and the boundary conditions are chosen accordingly as

$$\vartheta(\tau) = \vartheta_{10}, \ \sigma(\tau) = 0 \quad \text{for } \xi = +\infty$$
$$\vartheta(\tau) = 0, \ \sigma(\tau) = 1 \quad \text{for } \xi = 0$$

The stationary mode of flame propagation was considered as reached when $d \ln S(\tau)/d\tau \leq 0.001$. The distributions $\vartheta(\xi, \tau)$ and $\sigma_0(\xi, \tau)$ at this moment were taken as the initial distributions $\vartheta_0(\xi)$ and $\sigma_0(\xi)$.

3. It follows from the equations and the boundary conditions (2.12) that the solution depends on the parameters α_0, g_0, ϑ_{10}, Le, γ, and ν. As an example let us calculate the frequency characteristics of a flame with $\alpha_0 = 5$, $g_0 = 10.84$, $\vartheta_{10} = -8.67$, Le = 1, $\gamma = 1.4$, and a reaction order of $\nu = 1$ or 2. In particular, a hydrocarbon-air flame, for which E = 167.36 kJ/mole (40 kcal/mole), $T_{10} = 375°$ K, $T_{20} = 1865°$ K, and $\nu = 2$, is such a flame. The transition functions for these flames, calculated for four values of the pressure jump ($\Delta p = 0.005$, 0.01, 0.02, 0.04), are presented in Fig. 1. The value τ/τ_p is laid out along the abscissa, where τ_p is the relaxation time (it follows from the calculations that $\tau_p \approx 1$ and 0.5 for $\nu = 1$ and 2, respectively). The form of the transition functions is practically independent of the amplitude of the pressure disturbance. A slight difference is observed only at the starting times. Curve 1 corresponds to $\nu = 1$ and curve 2 to $\nu = 2$.

The initial sharp jump in the integral rate of heat release is explained by the increase in the chemical reaction rate because of the adiabatic increase in temperature at the time $\tau = 0$. The subsequent drop is caused by the restructuring of the front due to transport processes and is described approximately by an exponential function. The new stationary mode of flame propagation is determined by the adiabatic disturbances in the temperature and density of the initial mixture.

The amplitude–phase frequency characteristics of the flames calculated from the transition functions are presented in Fig. 2. The real and imaginary parts of the function $G(\Omega)$ are laid out along the coordinate axes. Curves 1, 2, and 3 pertain to a flame with $\nu = 1$ while curves 4, 5, 6, and 7 pertain to a flame with $\nu = 2$. Curve 1 is obtained for pressure jumps of $\Delta p = 0.005$ and 0.01 and the other curves are obtained for $\Delta p = 0.02$, 0.04, 0.005, 0.01, 0.02, and 0.04, respectively. The dimensionless frequencies are indicated near the plotted points ($\Omega = 2\pi f t_0$, where f is the dimensional frequency of the oscillations in Hz). A series of points which were calculated directly by numerical integration of Eqs. (2.12) for a harmonic pressure disturbance with amplitude $\delta p = 0.04$ is shown here by crosses for a flame with $\nu = 2$. Analogous calculations were made for $\delta p = 0.02$ and 0.08 with $\Omega = 100$.

The point G(100) for $\delta p = 0.02$ coincides with the corresponding point presented in Fig. 2 while for $\delta p = 0.08$ it is located somewhat below it. This is explained by the intensification in the nonlinearity of the oscillations in the rate of heat release with an increase in the amplitude of the pressure disturbances; at $\delta p = 0.08$ the ratio of the amplitude of the second harmonic of the rate of heat release to the amplitude of

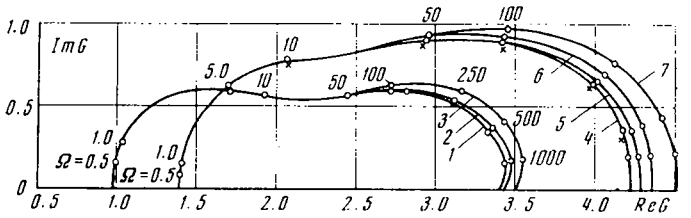

Fig. 2

the first harmonic is 0.2, while at $\delta p = 0.02$ and 0.04 the ratio is ~ 0.01. Hence it follows that with a decrease in the pressure disturbances the frequency characteristics obtained by the different methods converge. The good agreement of the results of the calculations by the two methods shows that the amplitude-phase frequency characteristic of a flame, which describes the stationary oscillations in the integral rate of heat release for a harmonic disturbance which has a small amplitude, can be determined from the transition function.

The form of the frequency characteristics indicates that resonance properties of the flame are practically absent. The modulus $G(\Omega)$ grows monotonically with an increase in frequency.

The oscillations in the integral rate of heat release are caused by the oscillations in the chemical reaction rate, which occur mainly because of the temperature variation in the reaction zone. As $\Omega \to 0$ the relaxation time of the front becomes much less than the period of the oscillations. The distributions of temperature and completeness of conversion which correspond to a front which propagates with constant velocity through a mixture with the current values of temperature and density are established almost instantly in the front. Because of this quasistationary nature of the flame propagation the temperature disturbances in the chemical reaction zone are equal to its adiabatic disturbances in the original mixture: $T_1' = (\gamma-1)\gamma^{-1}p'p_{10}^{-1}T_{10}$. Outside the reaction zone the temperature disturbances T_2' in the combustion products will be adiabatic, just as in the initial mixture: $T_2' = (\gamma-1)\gamma^{-1}p'p_{20}^{-1}T_{20}$ (in the equations for T_1' and T_2' all the values except γ are dimensional and $p_{10} = p_{20}$).

With an increase in the frequency the relaxation time of the front becomes considerably greater than the period of the oscillations. The processes of transport and heat release have less and less effect on the variation in gas temperature in an oscillation period, while the relation between the oscillations in pressure and temperature approaches more and more toward an isentropic relation. The temperature of the mixture in the reaction zone is close to the temperature T_{20} of the combustion products, hence the temperature disturbances in this zone will approach the value T_2' with an increase in frequency. Since $T_{20} > T_{10}$, the temperature disturbances in the chemical reaction zone at high frequencies are greater than at low frequencies. The reaction rate increases with an increase in temperature and accordingly the response of the flame at high frequencies is greater than at low frequencies.

It follows from Fig. 2 that the oscillations in the integral rate of heat release always lead the pressure oscillations. The phase shift between the oscillations in the rate of heat release and the pressure oscillations is explained by the phase shift between the temperature oscillations in the chemical reaction zone and the pressure oscillations. The appearance of such a phase shift results from the fact that the temperature variation at some fixed point of the gas due to pressure oscillations is accompanied by a simultaneous increase in the average temperature as it passes through the flame. Let us dwell on this point in more detail. It will now be convenient to convert to dimensional values for the dependent variables p, ρ, and u.

Let us consider an idealized model of a flame which consists of a surface of heat release and a Michelson temperature profile in front of it

$$T_0(q, t) = T_{10} + (T_{20} - T_{10}) \exp\left[-c_p m (q - mt) / \rho_{10}\chi_{10}\right]$$

where $q \geq mt$. We will assume that the mass propagation rate m of such a flame is constant. The surface of heat release passes through the point with the coordinate q at the time $t = q/m$. The temperature $T(q, t)$ at some fixed point of the gas with the coordinate q depends on the heating in the warmup zone of the flame and on the adiabatic compression or rarefaction in the pressure wave. If pressure disturbances are absent for $t < 0$ the temperature disturbance is

$$T'(q, t) = T(q, t) - T_0(q, t) = \frac{\gamma-1}{\gamma}\frac{1}{p_{10}}\int_0^t T_0(q, \zeta)\frac{dp}{d\zeta}\,d\zeta$$

Hence the temperature disturbance at the surface of heat release is

$$T_n'(t) = \frac{\gamma-1}{\gamma}\frac{1}{p_{10}}\int_0^t T_0(mt, \xi)\frac{dp'}{d\xi}\,d\xi$$

Let us change to the dimensionless independent variables τ and ξ, assuming that $u_0 = m/\rho_{10}$ in Eq. (2.3). Taking the pressure disturbance in the form $p' = \delta p \sin \Omega\tau$, after integration we obtain

$$T_n'(\tau) = \frac{\delta p}{p_{10}}\frac{\gamma-1}{\gamma}\left\{T_{10}\sin\Omega\tau + \frac{T_{20}-T_{10}}{1+\Omega^2}(\cos\Omega\tau + \Omega\sin\Omega\tau) - \frac{T_{20}-T_{10}}{1+\Omega^2}\Omega\exp(-\tau)\right\}$$

The last term is connected with the transitional process of establishing the oscillations $T_n'(\tau)$. As $\tau \to \infty$ it is reduced to zero. The first two terms describe the established temperature oscillations, which can be represented in the form

$$T_n'(\tau) = \frac{\delta p}{p_{10}}\frac{\gamma-1}{\gamma}T_{10}\left[\frac{1+\alpha_0^2\Omega^2}{1+\Omega^2}\right]^{1/2}\sin(\Omega\tau+\varphi)$$
$$\varphi = \operatorname{arc\,tg}\left[(\alpha_0-1)\,\Omega/(1+\alpha_0\Omega^2)\right],\quad \alpha_0 = T_{20}/T_{10} > 1 \qquad (3.1)$$

From this it follows that the temperature oscillations at the surface of heat release always lead the pressure oscillations ($\varphi > 0$), and the amplitude of the oscillations $T_n'(\tau)$ is a monotonically increasing function of the frequency.

We can give the rate of heat release at such an idealized flame front by the equation

$$F(q, t) = kp^{\nu}T^{-\nu}\exp(-E/RT)\,\delta(q-mt)$$

in which the dependence of the chemical reaction rate on the pressure and temperature is retained whereas the concentration of the initial material at the surface of heat release itself is reduced to zero. Here k is a proportionality coefficient and $\delta(q-mt)$ is the Dirac delta function. For the real and imaginary characteristics we obtain

$$\operatorname{Re}G_1(\Omega) = \frac{\nu}{\gamma} + \frac{\gamma-1}{\gamma}\left(\frac{g_0}{\alpha_0} - \vartheta_{10}\frac{\Omega^2}{1+\Omega^2}\right),\quad \operatorname{Im}G_1(\Omega) = -\frac{\gamma-1}{\gamma}\vartheta_{10}\frac{\Omega}{1+\Omega^2}$$

The amplitude–phase frequency characteristic described by these equations represents a semicircle located in the first quarter of the $G_1(\Omega)$ plane. As $\Omega \to \infty$ the values Re G_1 and Im G_1 approach the values calculated on the computer. The agreement of the general form of the function $G_1(\Omega)$ with the forms presented in Fig. 2 shows that the main reason for the dependence of the response of the flame on the frequency of the oscillations is the dependence of the temperature disturbances in the chemical reaction zone on the frequency. With an increase in the parameter $|\vartheta_{10}|$, which is proportional to the ratio of the time the gas remains in the entire flame front to the time it remains in the chemical reaction zone, the amplitude and phase characteristics of the flame become more frequency-dependent.

The frequency characteristics obtained can be used in a limited frequency region. The condition $\delta/\lambda \ll 1$ is equivalent to the condition $\Omega \ll 2\pi c_{10}/u_n\tau_n$, where δ is the width of the flame front, λ is the length of the sound wave, τ_n is the dimensionless time the gas remains in the flame front, c_{10} is the speed of sound, and u_n is the velocity of flame propagation. Let us estimate the region of admissible frequencies Ω for a flame with $\nu = 2$. According to the calculations $\tau_n = 0.3$ (at the leading boundary of the flame the completeness of conversion $\sigma = 0.01$, at the trailing front $\sigma = 0.99$); the characteristic value of the ratio $c_{10}/u_n = 600$ for a hydrocarbon–air mixture with an initial temperature of 373° K. Hence it follows that $\Omega \ll 1.3 \cdot 10^4$; with good approximation we have $\Omega \leq 10^3$.

4. Let us examine the effect of the physicochemical parameters on the frequency characteristics of flames having a second-order reaction. The form of the frequency characteristics depends mainly on the dimensionless parameters

$$\alpha_0 = T_{20}/T_{10},\quad g_0 = E/RT_{20},\quad \vartheta_{10} = (T_{10}-T_{20})\,E/RT_{20}^2$$

They cannot vary independently of one another. Therefore it is more convenient to analyze the dependence on the following dimensional parameters: the temperature of the initial mixture T_{10}, the calorific effect of the reaction $Q = cp(T_{20}-T_{10})/(1-h_{10})$, the activation energy E, and the composition of the mixture or the initial completeness of conversion h_{10}.

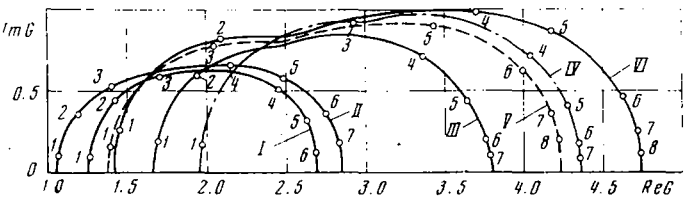

Fig. 3

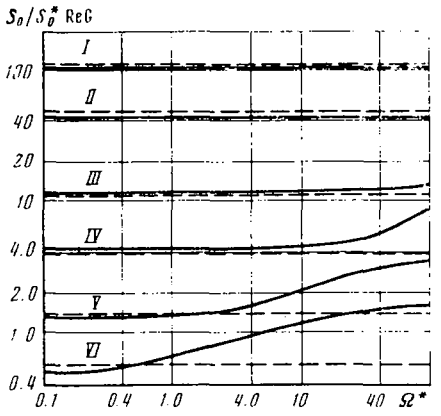

Fig. 4

Let us examine the effect of these parameters separately, considering the remaining dimensional parameters as constant. Let us select the flame front with the parameters taken in Part 3 as the initial, later called the standard, flame front. We will assume the preexponential factor k_i and the Lewis number Le to be identical for all the flames (Le = 1). An asterisk to the upper right of a character will mean that the value pertains to the standard flame.

Amplitude–phase frequency characteristics of flames which differ from the standard flame (curve V) by one of the parameters are presented in Fig. 3. The points on the curves are numbered. Point 1 corresponds to the frequency $\Omega = 1$, point 2 corresponds to 5, 3 to 10, 4 to 50, 5 to 100, 6 to 250, 7 to 500, and 8 to 1000. The same dimensional oscillation frequency f on these curves corresponds to different values of Ω: the connection between them is $\Omega^* = K\Omega$, where $K = t_0/t^*$.

A twofold decrease in activation energy (curve I) leads to a change in the dimensionless parameters $\alpha_0 = 5.0$, $g_0 = 5.42$, and $\vartheta_{10} = -4.34$ and to an increase in the integral rate of heat release S_0 and the normal velocity of flame propagation u_n. The results of numerical integration show that

$$S_0 = 85S_0^*, \quad u_n = 85u_n^*, \quad u_n = 4u_0, \quad K = 1800$$

A twofold increase in Q leads to an increase in the temperature of the combustion products $T_{20} = 3360°$ K (curve II). The dimensionless parameters for such a flame are $\alpha_0 = 9.0$, $g_0 = 6.0$, and $\vartheta_{10} = -5.36$; for the integral rate of heat release and the normal velocity of flame propagation we obtain

$$S_0 = 41.2S_0^*, \quad u_n = 20.6u_n^*, \quad u_n = 4.67u_0, \quad K = 162$$

A twofold increase in the temperature of the initial mixture (curve III) gives the following values of the parameters:

$$\alpha_0 = 3.0, \quad g_0 = 9.02, \quad \vartheta_{10} = -6.03, \quad S_0 = 7.28S_0^*, \quad u_n = 14.56u_n^*,$$
$$u_n = 3u_0, \quad K = 2.57$$

It is seen from the equations and boundary conditions (2.12) that the frequency characteristic $G(\Omega)$ does not depend on the average pressure and the initial completeness of conversion h_{10}. During the dilution of the initial mixture by the combustion products its thermophysical properties, which affect $G(\Omega)$, are changed.

Let us assume that the mixing-in of the hot combustion products takes place with the condition that the total specific enthalpy w of the mixture remains constant, $w = c_pT_{10} + Q(1 - h_{10})$ (curve IV). Then, taking $h_{10} = 0.25$, we obtain the increase in the initial temperature $T_{10} = 2T_{10}^*$. The parameters of this flame are:

$$\alpha_0 = 2.5, \quad g_0 = 10.84, \quad \vartheta_{10} = -6.5, \quad S_0 = 2.18S_0^*, \quad u_n = 5.8u_n^*,$$
$$u_n = 3.1\ u_0, \quad K = 6.27$$

If the cold combustion products are mixed in, the temperature of the initial mixture does not change, but the temperature of the combustion products drops because of the decrease in the caloricity of the mixture (curve VI). Taking $h_{10} = 0.25$ we obtain $T_{20} = 1429°$ K. The dimensionless parameters of such a front are $\alpha_0 = 4.0$, $g_0 = 12.2$, and $\vartheta_{10} = -10.15$, and for S_0 and u_n we obtain

$$S_0 = 0.35S_0^*, \quad u_n = 0.46u_n^*, \quad u_n = 3.1u_0, \quad K = 0.453$$

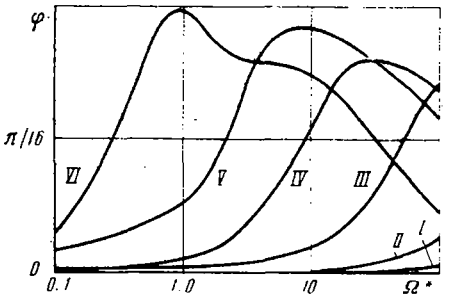

Fig. 5

If follows from Fig. 3 that variation in the physicochemical parameters of the flame within wide limits does not lead to a change in the overall form of the amplitude–phase frequency characteristic. The distribution of the markings of frequency Ω along the curves presented confirms the conclusion that the dependence of the amplitude and phase characteristics on the frequency increases with an increase in the modulus of the parameter ϑ_{10}.

5. The absolute values of the disturbances in the integral rate of heat release are important for an analysis of the stability of combustion. Let us examine the effect of changes in the physicochemical parameters on these values. The results of the appropriate calculations are presented in Fig. 4, which permits a comparison among the values of the disturbed integral rate of heat release for the flames examined above. The value $(S_0/S_0^*)\mathrm{Re}\,G$ is laid out along the ordinate. The ratio S_0/S_0^* is taken from the results of calculations for stationary fronts. The dimensionless frequencies calculated from the parameters of the standard front are laid out along the abscissa. The conversion coefficients K presented above are used for the change from the frequency Ω to the frequency Ω* (the designations of the curves in Figs. 4 and 5 are the same as in Fig. 3). It is seen from Fig. 4 that the greatest response occurs for flames with a high velocity of propagation. The responses of the flames under consideration calculated in the quasi-stationary approximation (dashed lines) are also presented in this figure. For stationary flame propagation the equation

$$S_0 = Q_1\rho_{10}u_n \tag{5.1}$$

is valid, where Q_1 is the caloricity of the mixture and u_n is the normal velocity of propagation of the flame. According to [7]

$$u_n^2 = \frac{2\nu!\,\chi_{20}k_i\rho_{10}^{\nu-2}(1-h_{10})^{\nu-1}}{c_p(T_{20}-T_{10})^{\nu+1}}\left(\frac{T_{10}}{T_{20}}\right)^{\nu}\left(\frac{RT_{20}^2}{E}\right)^{\nu+1}\exp\left(-\frac{E}{RT_{20}}\right) \tag{5.2}$$

The values of u_n calculated from this equation agree well with experiment. We assume that Eq. (5.1) is valid also for low frequencies Ω. Then, varying Eq. (5.1) for S_0 over all the variables except the completeness of conversion h_{10}, using an adiabatic dependence between the pressure, temperature, and density for the initial mixture, and keeping in mind the equality between the temperature disturbances in the chemical reaction zone with the temperature disturbances in the initial mixture for small Ω [Eq. (3.1)], i.e., $T_2' = T_1'$, we obtain the following equation for the frequency characteristic:

$$\mathrm{Re}\,G(\Omega) = -\frac{\nu}{2} + \frac{\gamma-1}{2\gamma}\left(\nu+2+\frac{E}{RT_{20}}\right)\frac{T_{10}}{T_{20}} \tag{5.3}$$

It is seen from Fig. 4 that the dashed lines calculated using this equation are located near the curves obtained by numerical integration: at low frequencies Ω the disturbances in the integral rate of heat release can be determined with good accuracy on the basis of the quasi-stationary approximation.

Let us clarify the region of dimensional frequencies for which this approximation is valid. For this one must know the characteristic time t_0, which can be found from Eq. (2.3) if the preexponential factor k_i is known. We choose it in such a way that the calculated value of u_n coincides with the experimental velocity of flame propagation U for the given parameters. From the results of the numerical integration we have $u_n = lu_0$ and for the standard front $l = 3.66$. We note that the value calculated for the standard front practically coincides with the value of u_n determined from Eq. (5.2). From a comparison of Eqs. (2.4) and (5.2) we have $u_n^2 = 2\nu!\,(T_{20}/T_{10})^{0.75}u_0^2$. Hence for the standard front $u_n = 3.66u_0$.

Taking $u_n = U$, for the determination of the dimensional frequency we obtain

$$f = (U^2 / 2\pi\varkappa_{10}l^2)\,\Omega$$

where $\varkappa_{10} = \chi_{10}/c_p\rho_{10}$ is the thermal diffusivity coefficient of the initial mixture. For the standard flame front with $T_{10} = 373°$ K the propagation velocity U, in the case of a benzene–air mixture for example, is 60 cm/sec and $\varkappa_{10} = 0.23$ cm^2/sec [8]. For these values we have $f = 185\Omega^*$ Hz.

The frequency dependence of the phase angle of the shift between the oscillations of the integral rate of heat release and the pressure oscillations is shown in Fig. 5. It is seen that this angle does not exceed

23°, so that the changes in the integral rate of heat release occur practically instantaneously with the pressure changes.

For all the flames examined [Re $G(\Omega)$ − Re $G(0)$]/Re $G(0) \leq 0.1$ for $\Omega \leq 3$ (see Fig. 3). Hence it follows that in a wide region of frequencies

$$0 \leqslant f \leqslant U^2 / 2\varkappa_{10} l^2$$

(depending on the parameters of the front $f \leq 500$–3000 Hz) the frequency characteristic can be calculated with sufficient accuracy for estimates of the self-triggering of the oscillations from Eq. (5.3) which was obtained in the quasi-stationary approximation.

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